Book Details

Title: Systems Analysis for Sustainable Engineering: Theory and Applications (Green Manufacturing & Systems Engineering)

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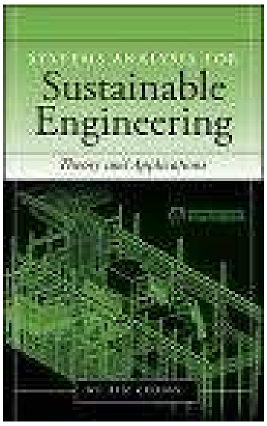
Description: Featuring a multidisciplinary approach, Systems Analysis for Sustainable Engineering: Theory and Applications provides a proven framework for applying systems analysis tools to account for environmental impacts, energy efficiency, cost-effectiveness, socioeconomic implications, and ecosystem health in engineering solutions. This pioneering work addresses the increased levels of sophistication embedded in many complex large-scale infrastructure systems and their interactions with the natural environment. After a detailed overview of sustainable systems engineering, the book covers mathematical theories of systems analysis, environmental resources management, industrial ecology, and sustainable design. Real-world examples highlight the methodologies presented in this authoritative resource.



Chapter 7: Integer Programming Models

7.6. **Linearization Techniques for Nonlinear Programming**

This section shows how binary variables can be used to model optimization problems involving piecewise linear functions. A piece-wise linear function is a function that consists of several straight-line segments that represent an approximation of a nonlinear function. Hence, nonlinear functions can be represented by IP formulation. LP can be used to maximize concave functions, or minimize convex functions, once they are replaced by piecewise-linear functions. Two common methods of doing this are illustrated in the following subsections. A third method allows the minimization of concave functions or the maximization of convex functions. This method requires the use of linear MIP optimization solution procedures in which some variables are forced to assume only integer values. [2] The following illustrate two MIP approaches. Method II is an extension of Method I that may exhibit higher convenience in model formulations.

7.6.1. **MIP Approach (I)**

Using this method, any separable function, whether convex, concave, or a combination of these shapes, can be approximated for inclusion in a linear integer programming model.

$$\text{Max or Min} \quad f(x) \cong f(a_1)z_1 + f(a_2)z_2 + f(a_3)z_3 + f(a_4)z_4 + \cdots$$

$$+ \; s_1x_1 + s_2x_2 + s_3x_3 + \cdots\cdots$$

$$= \sum_j [f(a_j)z_j + s_jx_j]$$

$$\text{s.t.} \quad a_1z_1 + a_2z_2 + a_3z_3 + a_4z_4 + \cdots\cdots + x_1 + x_2 + x_3 + \cdots\cdots = x$$

$$z_1 + z_2 + z_3 + \cdots\cdots = 1$$

$$x_j \le (a_{j+1} - a_j)z_j, \qquad \forall j$$

$$z_j = 0 \text{ or } 1, \qquad \forall j$$

7.6.2. **MIP Approach (II)**

Unlike Method I, Method II provides more accurate representation of nonlinear functions by using more integer variables and constraints. Note that Method II allows the value of f(x) to be defined in between two adjacent segment variables, which may not be achievable in Method II. Figure 7.5 can be used to illustrate Method II.

$$\text{Max or Min} \qquad f(x) \cong z_1 f(a_1) + z_2 f(a_2) + \cdots\cdots + z_n f(a_n)$$

$$\begin{aligned}
\text{s.t.} \qquad & z_1 \le y_1 \\
& z_2 \le y_1 + y_2 \\
& z_3 \le y_2 + y_3 \\
& \vdots \\
& \vdots \\
& z_{n-1} \le y_{n-2} + y_{n-1} \\
& z_n \le y_{n-1} \\
& y_1 + y_2 + \cdots\cdots + y_{n-1} = 1 \\
& z_1 + z_2 + \cdots\cdots + z_n = 1 \\
& x = z_1 a_1 + z_2 a_2 + \cdots\cdots + z_n a_n \\
& y_i = 0 \text{ or } 1, \quad \forall i \\
& z_i \ge 0, \quad \forall i
\end{aligned}$$

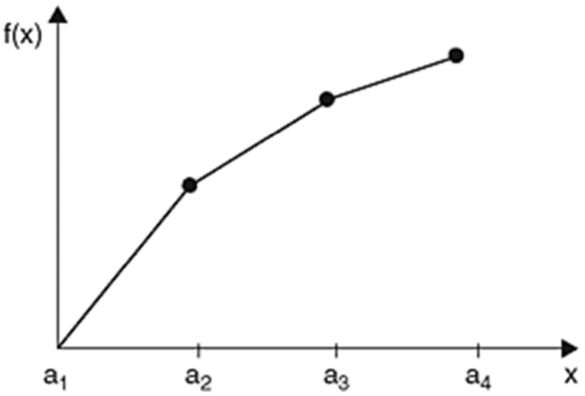


Figure 7.5. The linear approximation of a nonlinear function with respect to Method II.

Example 7.4 Linearize the following nonlinear programming model by Method II and solve the problem with LINDO.

$$\begin{aligned}
\max \qquad & 5x_1^2 + 11x_2^2 + 5x_3^{1/2} \\
\text{s.t.} \qquad & 3x_1 + x_2 + x_3 \le 20 \\
& -x_1 + x_2 \ge 4 \\
& x_2 + 1.5x_3 \le 10 \\
& x_1, x_2, x_3 \ge 0
\end{aligned}$$

Solution

STEP 1: Definition of Decision Variables and Parameters

Assume $f(x) = 5x_1^2, g(x) = 11x_2^2, h(x) = 4x_3^{0.5}$

If x_1, x_2, and x_3 are located in the interval [0, 10], then the values of f(x), g(x), and h(x) can be predicted in Table 7.2.

x	f(x)	g(x)	h(x)
0	0	0	0
2	20	44	5.66
4	80	176	8
6	150	396	9.79
8	320	704	11.31
10	500	1100	12.65

STEP 2: Model Formulation

Let $a_1 \sim a_5$, $b_1 \sim b_5$, $c_1 \sim c_5$ be the segment variables with regard to the f(x), g(x), and h(x). Then the model can be formulated as follows:

$$\text{Max} \quad Z = 20a_1 + 80a_2 + 150a_3 + 320a_4 + 500a_5 + 44b_1 + 176b_2 + 396b_3 + 704b_4 + 1100b_5 + 5.66c_1 + 8c_2 + 9.79c_3 + 11.31c_4 + 12.65c_5$$

s.t.

$$a_1 + a_2 + a_3 + a_4 + a_5 = 1$$
$$b_1 + b_2 + b_3 + b_4 + b_5 = 1$$
$$c_1 + c_2 + c_3 + c_4 + c_5 = 1$$
$$a_1 \le d_1$$
$$a_2 \le d_1 + d_2$$
$$a_3 \le d_2 + d_3$$
$$a_4 \le d_3 + d_4$$
$$a_5 \le d_4$$
$$b_1 \le e_1$$
$$b_2 \le e_1 + e_2$$
$$b_3 \le e_2 + e_3$$
$$b_4 \le e_3 + e_4$$
$$b_5 \le e_4$$
$$c_1 \le n_1$$
$$c_2 \le n_1 + n_2$$
$$c_3 \le n_2 + n_3$$
$$c_4 \le n_3 + n_4$$
$$c_5 \le n_4$$
$$d_1 + d_2 + d_3 + d_4 = 1$$
$$e_1 + e_2 + e_3 + e_4 = 1$$
$$n_1 + n_2 + n_3 + n_4 = 1$$
$$2a_1 + 4a_2 + 6a_3 + 8a_4 + 10a_5 - x_1 = 0$$
$$2b_1 + 4b_2 + 6b_3 + 8b_4 + 10b_5 - x_2 = 0$$
$$2c_1 + 4c_2 + 6c_3 + 8c_4 + 10c_5 - x_3 = 0$$
$$3x_1 + x_2 + x_3 \le 20$$
$$-x_1 + x_2 \le 4$$
$$x_2 + 1.5x_3 \le 10$$
$$d_1, d_2, d_3, d_4, e_1, e_2, e_3, e_4, n_1, n_2, n_3, n_4 \in \{0, 1\}$$

STEP 3: Preparation of the LINDO Code

$$\text{Max} \quad 20a1 + 80a2 + 150a3 + 320a4 + 500a5 + 44b1 + 176b2 + 396b3 + 704b4 + 1100b5 + 5.66c1 + 8c2 + 9.79c3 + 11.31c4 + 12.65c5$$

s.t.

$a1 + a2 + a3 + a4 + a5 = 1$

$b1 + b2 + b3 + b4 + b5 = 1$

$c1 + c2 + c3 + c4 + c5 = 1$

$a1 - d1 \leq 0$

$a2 - d1 - d2 \leq 0$

$a3 - d2 - d3 \leq 0$

$a4 - d3 - d4 \leq 0$

$a5 - d4 \leq 0$

$b1 - e1 \leq 0$

$b2 - e1 - e2 \leq 0$

$b3 - e2 - e3 \leq 0$

$b4 - e3 - e4 \leq 0$

$b5 - e4 \leq 0$

$c1 - n1 \leq 0$

$c2 - n1 - n2 \leq 0$

$c3 - n2 - n3 \leq 0$

$c4 - n3 - n4 \leq 0$

$c5 - n4 \leq 0$

$d1 + d2 + d3 + d4 = 1$

$e1 + e2 + e3 + e4 = 1$

$n1 + n2 + n3 + n4 = 1$

$2a1 + 4a2 + 6a3 + 8a4 + 10a5 - x1 = 0$

$2b1 + 4b2 + 6b3 + 8b4 + 10b5 - x2 = 0$

$2c1 + 4c2 + 6c3 + 8c4 + 10c5 - x3 = 0$

$3x1 + x2 + x3 \leq 20$

$-x1 + x2 \leq 4$

$x2 + 1.5x3 \leq 10$

end

INT d1

INT d2

INT d3

INT d4

INT e1

INT e2

INT e3

INT e4

INT n1

INT n2

INT n3

INT n4

STEP 4: List of LINDO Outputs

Z = 625.66	$x_1 = 3.66$	$x_2 = 7.0$	$x_3 = 2.0$	
$a_1 = 0.17$	$a_2 = 0.83$	$a_3 = 0$	$a_4 = 0$	$a_5 = 0$
$b_1 = 0$	$b_2 = 0$	$b_3 = 0.5$	$b_4 = 0.5$	$b_5 = 0$
$c_1 = 1$	$c_2 = 0$	$c_3 = 0$	$c_4 = 0$	$c_5 = 0$
$d_1 = 1$	$d_2 = 0$	$d_3 = 0$	$d_4 = 0$	
$e_1 = 0$	$e_2 = 0$	$e_3 = 1$	$e_4 = 0$	
$n_1 = 1$	$n_2 = 0$	$n_3 = 0$	$n_4 = 0$	