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Nonlinear pricing on transportation networks

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Abstract

Under nonlinear road pricing (or tolling), the price charged is not strictly proportional to the distance travelled inside a tolling area, the generalized travel cost is *not* link-wise additive, and finding a user equilibrium distribution is typically formulated as a complementarity problem. The latter is a difficult problem to solve in mathematical programming. In this paper, we use piecewise linear functions to determine tolls and show that finding a user equilibrium distribution with such functions can be formulated as a convex optimization problem that is based on path flows and solvable by traditional algorithms such as simplicial decomposition. For area-based and two-part pricing schemes, the tolling function consists of only one linear piece and finding a user equilibrium distribution reduces

to a convex optimization problem formulated in terms of link flows and solvable by any software for linearly constrained convex programs.

To find an optimal pricing scheme, e.g., one that maximizes the social benefit, we formulate the problem as a mathematical program with equilibrium constraints, an optimization problem that is generally non-convex and difficult to solve. However, it is possible to use search algorithms to find an optimal scheme because the number of parameters in our piecewise linear function is few. To illustrate, we use a coordinate search algorithm to find an optimal two-part pricing scheme for a small network with randomly generated data.

Highlights

► This paper assumes that the amounts of toll that users pay varies nonlinearly with the distance they travel inside a tolling area. ► When the toll pricing function is piecewise linear, the tolled UE problem is a convex program. ► For two-part and area-based pricing, the UE conditions and problem can be stated using link flows.

Keywords

- Road pricing;
- Congestion;
- Tolls;
- Equilibrium;
- Planning

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1. Introduction

Nonlinear pricing generally refers to a case in which the price or tariff is not strictly proportional to the quantity purchased. Economists have been studying such pricing since the discussion of its manifestations in Dupuit (1894) and the later categorization of the phenomenon in Pigou (1920). Today, nonlinear pricing is prevalent in many industries. For example, railroad tariffs generally depend on the weight, volume, and distance of each shipment. However, those using full-cars and/or over long distances often receive discounts. The price per kilowatt-hour of electricity is different for different types of users. Heavy users during peak hours generally pay higher rates. Airlines routinely offer discount tickets for advance purchase, with non-cancellation restriction, and in competitive markets. In each of these examples, the average price paid per unit varies depending on characteristics of the purchase such as its size, time of usage, and restrictions.

In practice, road pricing is often nonlinear. The tolls in, e.g., Singapore (Menon et al., 1993), London (Santos and Shaffer, 2004), and Stockholm (Stockholmsforsoket, 2006) are not proportional to the distance travelled inside the tolling areas. In Stockholm, tolls are also not proportional to the number of times a user enters the tolling area. The amount of tolls paid on a given day is limited to

SEK 60. After paying this maximum amount, users can freely enter the tolling area for the rest of the day. For its congestion charge, London offers monthly and annual passes to frequent users at an approximately 15% discount. Similarly, the Dulles Greenway’s VIP Frequent Rider Program gives rebates to users with high mileage. During phase I of its Value Pricing Project on Interstate 15, San Diego sold $50 monthly permits that allow single occupancy vehicles to use lanes reserved for high occupancy vehicles. (During phase II, the permits were replaced by tolls.)

Despite its widespread use, the literature on nonlinear road pricing is limited. De Borger (2001) proposes a discrete choice model to study optimal two-part tariffs in the presence of externalities. In their nonlinear pricing study, Wang et al. (2011) consider three questions: which nonlinear pricing scheme (among the five they consider) is most profitable, how does the most profitable choice depend on congestion, and does usage-only pricing necessarily denominate other nonlinear schemes if congestion is severe? Both [De Borger, 2001] and [Wang et al., 2011] opine that nonlinear pricing has been largely overlooked in the literature. Separate from the previous two papers, Gabriel and Bernstein (1997a) formulate the problem of finding a user equilibrium (UE) distribution on general road networks (or, more simply, the UE problem) when travel costs are not link-wise additive as a nonlinear complementarity problem or NCP. In their formulation, one component of the path travel cost is a nonlinear function of its travel distance. To solve their UE problem, Gabriel and Bernstein (1997a) propose an algorithm based on nonsmooth equations and sequential quadratic programming (see also Gabriel and Bernstein, 1997b). Lo and Chen (2000) consider a similar problem and convert their NCP into an unconstrained optimization problem based on a merit function. More recently, Agdeppa et al. (2007) modify the model in Gabriel and Bernstein (1997a) by introducing a disutility function and formulate the problem as a monotone mixed complementarity problem instead. [Maruyama and Harata, 2006] and [Maruyama and Sumalee, 2007] propose an algorithm for area-based pricing, one form of nonlinear pricing. The authors of the last two papers observe that area-based pricing is not link-wise additive and it may be reasonable and intuitive to conclude from this that no equilibrium condition based on link flows exists. However, as demonstrated below, this conclusion is incorrect.

This paper considers nonlinear pricing in the context of managing travel demand, reducing congestion, and, perhaps, lessening the environmental impact in a tolling area. Although it is common to assume that a tolling area consists of connected roads or roads in a connected geographical area, such an assumption is unnecessary. For example, a tolling area can consist of not necessarily connected roads or highways that are under the jurisdiction of a single entity (a government agency or private company). It is also possible to let the tolling area be the entire road network and every road user must pay tolls. Doing so reduces our problem to the one addressed in Gabriel and Bernstein (1977a).

In this paper, the amount of toll that users pay, $T(\ell)$, varies nonlinearly with ℓ, the distance travelled inside the tolling area. (Henceforth, $T(\ell)$ is also referred to as the tolling or pricing function.) We assume that $T(\ell)$ is piecewise linear and the number of linear pieces is two or less. As observed in Wilson (1993), a piecewise linear function with a small number of linear pieces is easier to understand, thus more practical, and can realize most of the advantages of general nonlinear pricing functions. As demonstrated below, the UE problem with piecewise linear pricing functions reduces to an optimization problem that is similar to the standard UE problem (see, e.g., Florian and Hearn, 2003) and solvable by well-known algorithms such as simplicial decomposition. For area-based and two-part pricing schemes, both user equilibrium conditions and the UE problem can be formulated in term of link flows despite the fact that the generalized cost is not link-wise additive. Solving the link-based UE problem eliminates the need to maintain information about individual paths and typically requires less computational resources. In fact, the UE problem with area-based and two-part pricing schemes can be solved by any software for linearly constrained convex programs.

To our knowledge, there has been little or no attempt to find an optimal nonlinear pricing scheme for a general road network. To find an optimal scheme, De Borger (2001) assumes that the travel demand is measured in kilometres without an explicit road network. Similarly, Wang et al. (2011) consider a network with only one link. In this paper, we formulate the problem of finding a nonlinear pricing scheme that, e.g., maximizes the social benefit as a mathematical program with equilibrium constraints. We demonstrate that such a problem can be solved using a search algorithm when the tolling function is piecewise linear.

For the remainder, Section 2 describes the pricing functions considered in this paper. Section 3 defines our notation and states path-based UE conditions for later reference. Section 4 formulates the UE problem in terms of path flows and modifies simplicial decomposition to find a UE flow-demand pair under our nonlinear pricing functions. Section 5 states link-based UE conditions and discusses when these conditions are equivalent to those based on paths. Section 6 presents a search algorithm for finding optimal pricing parameters, e.g., those that maximize the social benefit. Finally, Section 7 studies numerical results from a small road network with randomly generated data and Section 8 concludes the paper. To illustrate the simplicity of using link flows, the Appendix gives a version of the Frank-Wolfe algorithm (a well-known algorithm for linearly constrained convex programs − see Frank and Wolfe, 1956) for solving the UE problem with two-part pricing.

2. Nonlinear pricing functions

The tolling function, $T(\ell)$, in this paper is assumed for simplicity to be in units of time and of the form:

$$T(\ell) = \begin{cases} T^{min}(\ell) \text{ or } T^{max}(\ell), & \ell > 0 \\ 0, & \ell \leqslant 0 \end{cases}$$

where $Tmin(\ell) = \min\{\beta_1 + \mu_1 \ell, \beta_2 + \mu_2 \ell\}$ and $T^{max}(\ell) = \max\{\beta_1 + \mu_1 \ell, \beta_2 + \mu_2 \ell\}$. Recall that ℓ is the distance travelled inside the tolling area. (Herein, distances are measured in miles and we refer to a rate or fee based on miles travelled as a "VMT fee", where VMT is an abbreviation for "vehicle-mile travelled.") In both $Tmin(\ell)$ and $Tmax(\ell)$, μ_1 and μ_2 are nonnegative VMT fees. Typically, β_1 and β_2 are nonnegative. However, one may be negative to reproduce some tolling functions in practice more accurately. (See the discussion about three-part tariffs below.)

Both $Tmin(\ell)$ and $Tmax(\ell)$ are piecewise linear functions with two linear pieces. Although the number of linear pieces can be larger, i.e., $Tmin(\ell) = \min\{\beta_1 + \mu_1\ell, \cdots, \beta n + \mu n\ell\}$ and $Tmax(\ell) = \max\{\beta_1 + \mu_1\ell, \cdots, \beta n + \mu n\ell\}$, where $n \geqslant 2$, we set $n = 2$ in this paper for two reasons. First, the results for $n = 2$ can be extended to the cases with larger n without much difficulty. As cautioned in Wilson (1993), the second reason is that large n is often *not* practical. Pricing functions with many linear pieces generally result in tolling schemes too complex for motorists to understand and respond properly. Moreover, pricing functions with only a few linear pieces can typically capture most of the benefits offered by those with many.

When β_1, μ_1, β_2 and μ_2 are chosen appropriately, $T^{min}(\ell)$ and $T^{max}(\ell)$ capture common nonlinear pricing functions in the economics and road pricing literature (see, e.g., [Wilson, 1993] and [Wang et al., 2011]). Fig. 2.1 displays tolling functions based on *Tmin*(ℓ). In case (a), the VMT fee for a longer distance (μ_2) is smaller than the one for a shorter distance (μ_1), i.e., heavy road users receive discounts. Case (b) allows users to either pay a VMT fee at a rate μ_1 or a fixed fee, β_2, for unlimited travel inside the tolling area. The former is more economical when the travel distance is sufficiently short, i.e., less than the point where $\mu_1\ell = \beta_2$. Although both cases may be suitable for many industries, it is not clear that they would be adopted for congestion mitigation.

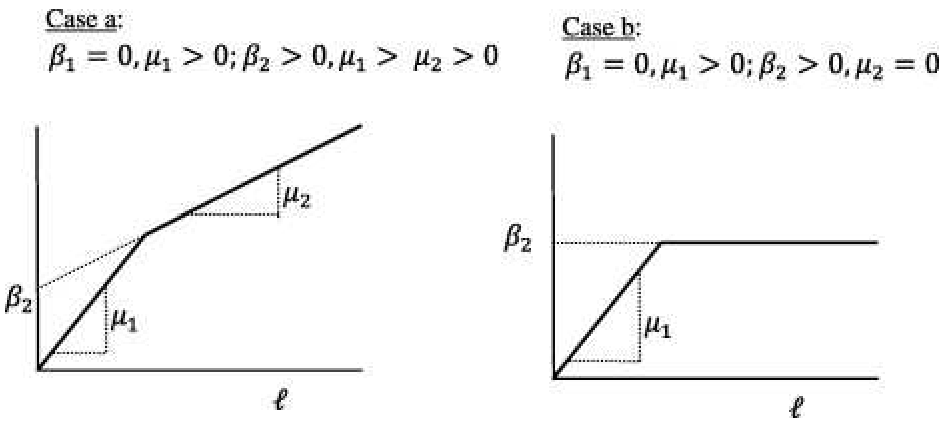

Fig. 2.1. Pricing functions based on $T^{min}(\ell)$.

For the pricing functions based on $T^{max}(\ell)$ in Fig. 2.2, case (a) requires users to pay two fees. One is an access fee (β_1) and the other is a VMT fee (μ_1). Economists commonly refer to this form of pricing as a two-part tariff or pricing scheme. Similarly, the function in case (b) also consists of an access and VMT fee. However, the latter only applies when the travel distance exceeds a threshold, a point where $\beta_1 + \mu_1\ell = \beta_2$. (When β_2 and μ_1 are fixed, β_1 may need to be negative to achieve a desired threshold value.) In economics, some refer to case (b) as a three-part tariff. Instead of giving discounts to heavy users, case (c) discourages heavy road usage by charging a higher VMT fee (μ_1) when the travel distance exceeds a threshold, a point where $\beta_1 + \mu_1\ell = \beta_2 + \mu_2\ell$. Finally, the pricing function for case (d) is suitable for area-based pricing (see, e.g., Maruyama and Sumalee, 2007), a tolling scheme under which users can enter and use the tolling area as often and as much as they like during a specified period after paying an access fee, β_1. (Area-based pricing is different from cordon pricing (see, e.g., Zhang and Yang, 2004). For the latter, users generally pay a fee each time they enter the tolling area.) In addition to those shown in the two figures, setting β_1, β_2, and μ_2 to zero reduces $T(\ell)$ to linear pricing, i.e., $T(\ell) = \mu_1\ell$.

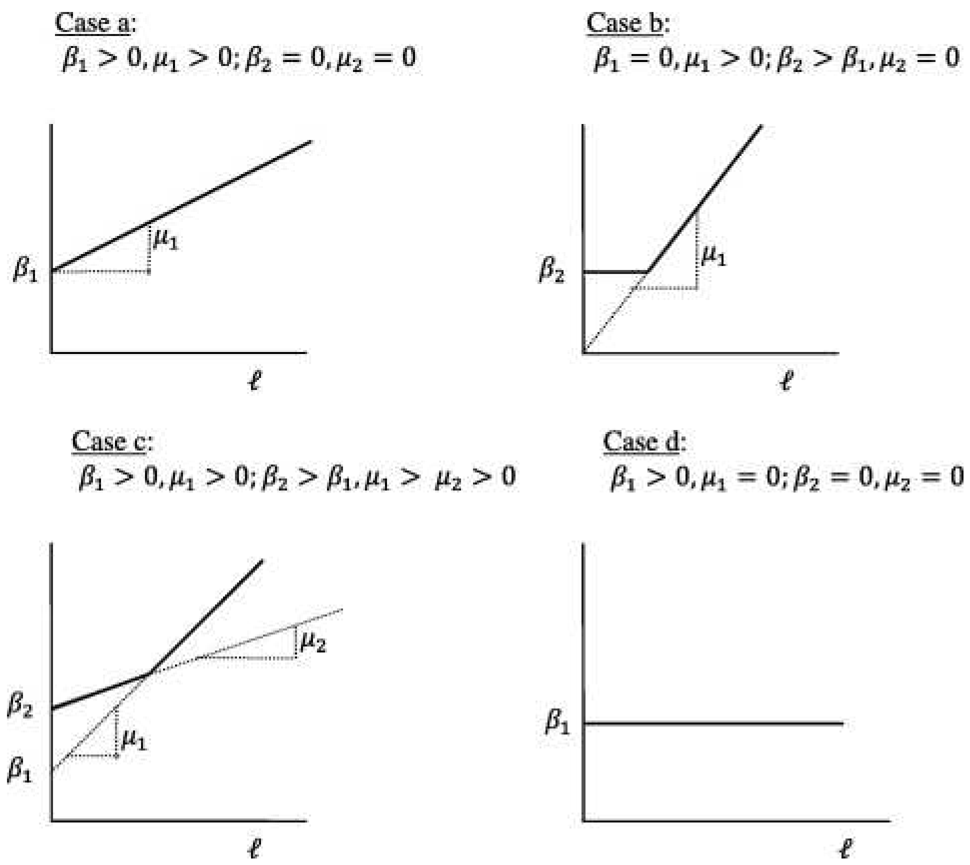

Fig. 2.2. Pricing functions based on $T^{max}(\ell)$.